

How robust is a 2SC quark matter phase under compact star constraints?

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Abstract

We study the phase structure and equation of state for two-flavor quark matter at low temperature under compact star constraints within a nonlocal chiral quark model. We find that the occurrence of a two-flavor color superconducting (2SC) phase is sensitive to variations of both the formfactor of the interaction and the ratio η between the coupling constants in the diquark and the scalar meson channels. Our study suggests that for standard values of the coupling ratio $0.5 < \eta < 0.75$ either the 2SC phase does not occur (Gaussian formfactor) or it exists only in a mixed phase with normal quark matter (NQ-2SC) with a volume fraction less than 20 – 40 %, occurring at high baryon chemical potentials $\mu_B > 1200$ MeV and most likely not relevant for compact stars. We also present the relevant region of the phase diagram for compact star applications and obtain that no gapless 2SC occurs at low temperatures.

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1 Introduction

One of the challenges of strong interaction physics nowadays is to understand the phase diagram of quantum chromodynamics (QCD). Particular interest is devoted to the high density and low temperature region because of possible applications for the physics of compact stars. These investigations became very exciting since it has been reported that the energy gaps of color superconducting quark matter could be as big as $\Delta \sim 100$ MeV [1,2] and thus being of the same order as the constituent quark masses dynamically generated in the chiral symmetry breaking transition. Gaps this large could not only have a direct influence on the quark matter equation of state (EoS) but, due to the BCS relation $T_c = 0.57\Delta$ critical temperatures would imply that quark matter if formed early in the protoneutron star evolution should be color superconducting.

Dense and cold matter has been broadly studied for the flavor symmetric case (for reviews, see [3,4]). From first principles, the limit of asymptotic densities can be studied and it was found that the color-flavor-locking (CFL) phase, in which quarks of all three colors and flavors participate in the condensation, is energetically favorable (see [5]). Using the simple Nambu-Jona-Lasinio model [6], it was found that for intermediate densities the 2SC phase was favored [7]. Therefore, a typical phase diagram calculated for the flavor symmetric case shows the low temperature and high density domain being in one of the color superconducting quark matter phases.

However, for compact star applications the situation is much more complex: equilibrium w.r.t. weak interactions (β -equilibrium), color and charge neutrality impose additional constraints to dense matter. Alford and Rajagopal pointed out the difficulty of the 2SC phase to achieve charge neutrality and therefore they claimed that this phase would not be realized in compact stars [8]. Without solving the gap equation, they treated the strange quark mass as a parameter in their calculations and concluded that the CFL phase would be favorable against 2SC.

More recently, Steiner, Reddy and Prakash [9] found within an NJL model calculation that the 2SC phase may be realized only in a very small region of the phase diagram at lower densities which eventually would be covered by the hadronic phase if it was properly take into account. Again, the 2SC phase seemed not to be reliable, basically because the charge neutrality condition imposes a strong constraint to the quark chemical potentials.

The discussion of the existence of a mixed phase of 2SC quark matter and normal quark matter (NQ-2SC) by increasing the asymmetry between the up and down quarks number densities was first performed by Bedaque [10]. Within the

NJL model, a detailed study of possible neutral mixed phases in β -equilibrium has been performed by Neumann, Buballa and Oertel [11] considering global neutrality between the phases. In accordance with the earlier expectations, these authors showed that at very low densities - where the model is probably to fail - the normal quark matter component is dominant, at intermediate ones the 2SC phase and at still higher densities, the CFL phase. Moreover, the latter phase and the strange quarks is supposed to appear at very high densities (nearly above 1500 MeV [12,13]) that barely reached in the very inner core of compact stars and thus leading to no observable effects.

Unfortunately, in the intermediate density region we have no rigorous calculations and we have to rely on phenomenological models suitable, however, for the investigation of qualitative features of the phase diagram. The above mentioned and widely used NJL has the great advantage that chiral symmetry breaking and diquark condensation are treated at the same mean-field level as phase transitions of an interacting field theory.

Besides well-known limitations of the NJL-type models (no confinement, non-renormalizable) there is some arbitrariness in the choice of the model parameters even if the strategy is followed that vacuum properties of low-lying hadrons should be reproduced (e.g. the pion mass, pion decay constant and the chiral condensate), see e.g. [14,13] for reviews.

In this work we use a chiral quark model that can represent the nonlocality of the quark interactions in a more realistic way via formfactor functions. Furthermore, we vary the softness of these functions and analyze its consequences in the phase structure. We investigate the parameter regime and obtain that no pure 2SC itself does occur under compact star constraints for low temperatures (below 25 MeV). We also study the relevant region of the phase diagram for compact star applications and obtain that neither gapless 2SC (g2SC [15]) exists; its occurrence is limited to a narrow window for higher temperatures (above 40 MeV) before the second order phase transition to the deconfined phase takes place.

For rather strong coupling constants in the diquark channel ($\eta \geq 0.86$) a mixed phase NQ-2SC is likely to occur in the interior of compact stars. For particular values of η the corresponding equation of state has been already successfully used for hybrid star configurations, explaining the mass-radius relation of very compact objects like RX J185635-3754 [17] or a huge release of energy in the evolution of a protoneutron star with trapped antineutrinos [18].

Nevertheless, for weak and intermediate coupling constants (even for $\eta = 0.75$, the value suggested by the Fierz transformation of a massive gluon exchange or two-flavor instanton induced interaction [13]), the 2SC phase is unlikely to occur. If this is the case, this result could have important consequences for the

neutron star phenomenology: if 2SC does not occur inside compact stars, then other patterns of diquark pairing satisfying color and charge neutrality can become energetically more favorable than normal quark matter. Examples are spin-1 condensates, e.g. color-spin-locking (CSL) phases [19,20,21]. The influence of the small gaps (10 – 100 keV) that are expected for these phases on compact star cooling has recently been investigated and they could realize the appropriate cooling phenomenology [22].

2 Quark matter under compact stars constraints

2.1 Two flavor quark matter with color superconductivity

We use a nonlocal chiral model for quark matter in the 2SC phase to study dense neutral matter in compact stars following [17]. Formfactor functions $g(p)$ model the nonlocality of the quark interaction in the momentum space. We assume that this four fermion interaction is instantaneous and therefore the formfactors depend only on the modulus of the three-momentum $p = |\vec{p}|$. G_1 and G_2 are the coupling constants in the scalar meson and the diquark channels, respectively. While G_1 is fixed together with the current quark mass and range parameters of the formfactors (see below) from hadron observables, G_2 is a free parameter of the approach which we vary using the parameter $\eta = G_2/G_1$.

In the mean field approximation the grand canonical thermodynamic potential is a function of the temperature T and of the chemical potentials μ_{fc} for the quark with flavor f ($f \in \{u, d\}$) and color c ($c \in \{r, b, g\}$) given by

$$\Omega_q(\{\mu_{fc}\}, T) = -T \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{1}{2} \text{Tr} \ln \left(\frac{1}{T} \tilde{S}^{-1}(i\omega_n, \vec{p}) \right) + V - \Omega_{\text{vac}} , \quad (1)$$

where the effective potential

$$V = \frac{\phi_u^2 + \phi_d^2}{8 G_1} + \frac{\Delta^2}{4 G_2} \quad (2)$$

is a function of the order parameters of the theory: the mass gap ϕ_f and the diquark gap Δ . The constant Ω_{vac} is chosen such that the pressure of the physical vacuum vanishes. The Matsubara frequencies for fermions are given by $\omega_n = (2n + 1)\pi T$ and the Nambu-Gorkov inverse quark propagator can be

obtained as

$$\tilde{S}^{-1}(p_0, \vec{p}) = \begin{pmatrix} \not{p} - \hat{M} - \hat{\mu}\gamma_0 & \Delta\gamma_5\epsilon\epsilon^b g(p) \\ -\Delta^*\gamma_5\epsilon\epsilon^b g(p) & \not{p} - \hat{M} + \hat{\mu}\gamma_0 \end{pmatrix}, \quad (3)$$

where $\hat{\mu} = \text{diag}(\mu_{fc})$ is the chemical potential matrix and the elements of $\hat{M}(p) = \text{diag}(M_f(p))$ are the dynamical masses of the quarks given by $M_f(p) = m_f + g(p)\phi_f$. Here, γ_5 is the usual matrix in the Dirac space and $(\epsilon)^{ik} \equiv \epsilon^{ik}$ and $(\epsilon^b)^{\alpha\beta} \equiv \epsilon^{\alpha\beta b}$ are antisymmetric tensors in the flavor and color spaces respectively.

The nonlocality of the interaction between the quarks in both channels quark-antiquark ($q\bar{q}$) and quark-quark (qq) is implemented via the same formfactor functions $g(p)$. In our calculations we use the Gaussian (G), Lorentzian (L) and cutoff (NJL) formfactors defined as

$$g_G(p) = \exp(-p^2/\Lambda_G^2), \quad (4)$$

$$g_L(p) = [1 + (p/\Lambda_L)^2]^{-1}, \quad (5)$$

$$g_{\text{NJL}}(p) = \theta(1 - p/\Lambda_{\text{NJL}}). \quad (6)$$

The parameter sets (quark mass $m = m_u = m_d$, coupling constant G_1 , interaction range Λ) for the above formfactor models (see Tab. 1) are fixed by the pion mass $m_\pi = 140$ MeV, pion decay constant $f_\pi = 93$ MeV and the mass gap $\phi = \phi_u = \phi_d = 330$ MeV at $T = \mu = 0$ [23].

In order to describe asymmetric two flavor quark matter [24] in a neutral phase [25] we introduce the baryon chemical potential μ_B , the chemical potential for the isospin asymmetry μ_I and the chemical potential for the color asymmetry μ_8 . These chemical potentials are conjugate to the conserved quantities in dense matter in neutron stars: baryon number density, electric and color charge density, respectively (see next subsection). We express $\{\mu_{fc}\}$ in terms of $\{\mu_B, \mu_I, \mu_8\}$:

$$\mu_{ur} = \mu_{ug} = \frac{1}{3}(\mu_B + 4\mu_I + \mu_8), \quad (7)$$

$$\mu_{dr} = \mu_{dg} = \frac{1}{3}(\mu_B - 2\mu_I + \mu_8), \quad (8)$$

$$\mu_{ub} = \frac{1}{3}(\mu_B + 4\mu_I - 2\mu_8), \quad (9)$$

$$\mu_{db} = \frac{1}{3}(\mu_B - 2\mu_I - 2\mu_8). \quad (10)$$

For the 2SC pairing pattern we assume that $\Delta_c = g(p)\Delta(\delta_{c,r} + \delta_{c,g})$ in which

the red (r) and the green (g) quarks are paired. The dispersion relation of the unpaired color (blue) is $E_f(p) = \sqrt{p^2 + M_f^2(p)}$.

As has been shown in [26] for the 2SC phase the relation $\phi_u = \phi_d = \phi$ holds and therefore $E_u(p) = E_d(p) = E(p)$ so that the quark thermodynamic potential can be written more explicitly as

$$\begin{aligned}
\Omega_q(\mu_B, \mu_I, \mu_8, T) + \Omega_{vac} &= \frac{\phi^2}{4G_1} + \frac{\Delta^2}{4G_2} \\
&- \frac{1}{\pi^2} \int_0^\infty dp p^2 \{ \omega \left[\epsilon_b \left(E(p) - \frac{1}{3}(\mu_B + \mu_I) + \frac{2}{3}\mu_8 \right) - \mu_I, T \right] + \\
&\omega \left[\epsilon_b \left(E(p) + \frac{1}{3}(\mu_B + \mu_I) - \frac{2}{3}\mu_8 \right) - \mu_I, T \right] + \\
&\omega \left[\epsilon_b \left(E(p) - \frac{1}{3}(\mu_B + \mu_I) + \frac{2}{3}\mu_8 \right) + \mu_I, T \right] + \\
&\omega \left[\epsilon_b \left(E(p) + \frac{1}{3}(\mu_B + \mu_I) - \frac{2}{3}\mu_8 \right) + \mu_I, T \right] \} \\
&- \frac{2}{\pi^2} \int_0^\infty dp p^2 \{ \omega \left[\epsilon_r \left(E(p) - \frac{1}{3}(\mu_B + \mu_I) - \frac{1}{3}\mu_8 \right) - \mu_I, T \right] + \\
&\omega \left[\epsilon_r \left(E(p) + \frac{1}{3}(\mu_B + \mu_I) + \frac{1}{3}\mu_8 \right) - \mu_I, T \right] + \\
&\omega \left[\epsilon_r \left(E(p) - \frac{1}{3}(\mu_B + \mu_I) - \frac{1}{3}\mu_8 \right) + \mu_I, T \right] + \\
&\omega \left[\epsilon_r \left(E(p) + \frac{1}{3}(\mu_B + \mu_I) + \frac{1}{3}\mu_8 \right) + \mu_I, T \right] \} , \tag{11}
\end{aligned}$$

where following the notation in [17]

$$\epsilon_c(\xi) = \xi \sqrt{1 + \Delta_c^2/\xi^2} , \tag{12}$$

$$\omega[x, T] = T \ln \left[1 + \exp \left(-\frac{x}{T} \right) \right] + \frac{x}{2} . \tag{13}$$

The factor 2 in the last integral in (11) comes from the degeneracy of the red and green colors ($\epsilon_r(\xi) = \epsilon_g(\xi)$).

The total thermodynamic potential Ω contains the quark contribution Ω_q and the contribution Ω^{id} of the leptons l that are treated as a massless, ideal Fermi gas:

$$\Omega(\mu_B, \mu_I, \mu_8, \mu_l, T) = \Omega_q(\mu_B, \mu_I, \mu_8, T) + \sum_l \Omega^{id}(\mu_l, T) . \tag{14}$$

The conditions for the local extremum of Ω_q correspond to coupled gap equations for the two order parameters ϕ and Δ , $\partial\Omega/\partial\phi = \partial\Omega/\partial\Delta = 0$. The global minimum of Ω_q represents the state of thermodynamic equilibrium from which the equation of state can be obtained by derivation.

2.2 Beta equilibrium, charge and color neutrality

The stellar matter in the quark core of compact stars consists of $\{u, d\}$ quarks and leptons $\{e, \nu_e, \bar{\nu}_e, \mu, \nu_\mu, \bar{\nu}_\mu\}$ under the conditions of

- β -equilibrium:

$$\begin{aligned} d &\longleftrightarrow u + e + \bar{\nu}_e \\ d &\longleftrightarrow u + \mu + \bar{\nu}_\mu \\ u + e &\longleftrightarrow d + \nu_e , \end{aligned} \tag{15}$$

which in terms of chemical potentials reads ($\mu_{\bar{\nu}_e} = -\mu_{\nu_e}$, $\mu_{\bar{\nu}_\mu} = -\mu_{\nu_\mu} \simeq 0$)

$$\mu_e + \mu_{\bar{\nu}_e} = \mu_\mu = -2\mu_I , \tag{16}$$

- charge neutrality:

$$\frac{2}{3}n_u - \frac{1}{3}n_d - n_e - n_\mu = 0 , \tag{17}$$

which could also be written as

$$n_B + n_I - 2n_e - n_\mu = 0 \tag{18}$$

and

- color neutrality:

$$n_8 = \frac{1}{3}(n_r + n_g - 2n_b) = 0 . \tag{19}$$

Here the number densities n_j are defined in relation to the corresponding chemical potentials μ_j as

$$n_j = -\left.\frac{\partial\Omega}{\partial\mu_j}\right|_{\phi_0, \Delta_0, T} , \tag{20}$$

where the index j denotes the particle species and $n_f = \sum_c n_{fc}$ and $n_c = \sum_f n_{fc}$. The solution of the color neutrality condition shows that μ_8 is about 5-7 MeV in the region of relevant densities ($\mu_B \simeq 900 - 1500$ MeV). Since the isospin asymmetry is independent of μ_8 we consider $\mu_8 \simeq 0$ in our following calculations.

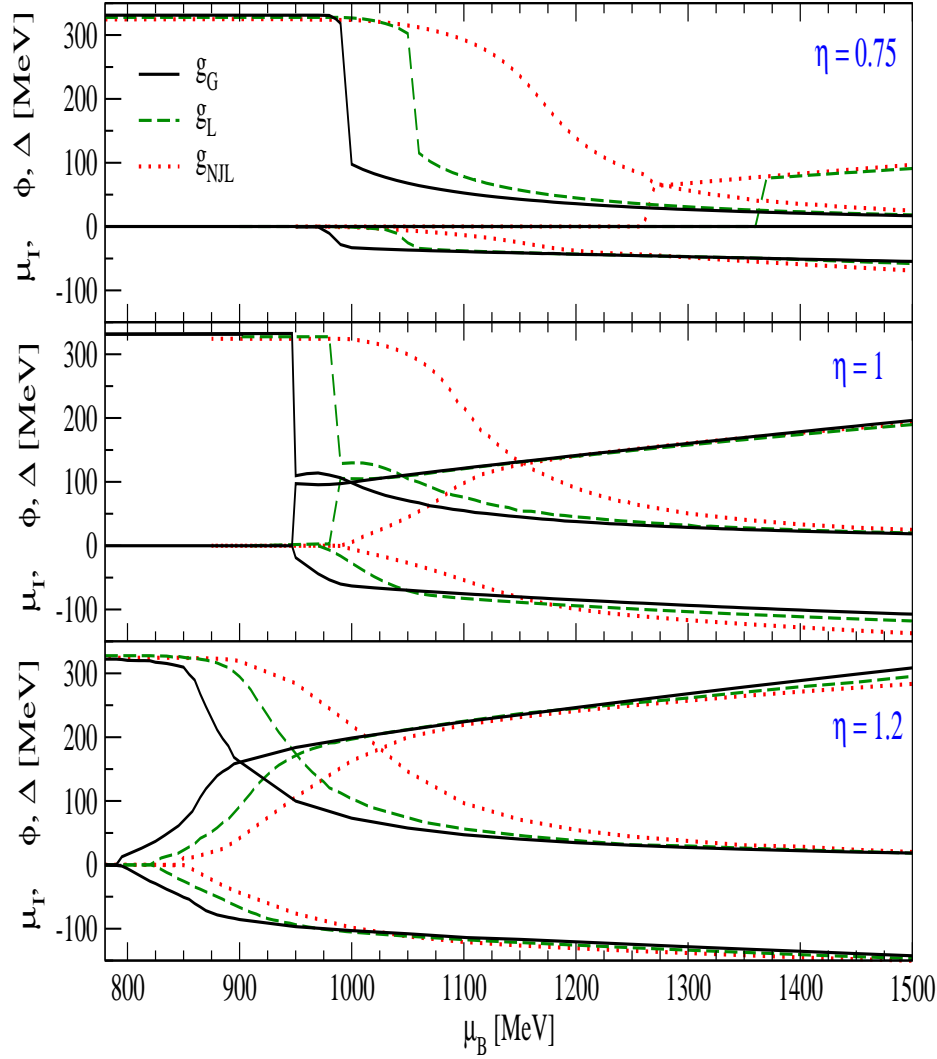


Fig. 1. Gap equation solutions for neutral matter in β -equilibrium for different strengths η of the coupling constant in the diquark channel at $T = 0$. In the upper part of each plot, the order parameters ϕ and Δ are displayed as a function of the baryon chemical potential. In the lower part, the corresponding asymmetry is shown. The results are calculated for three different formfactors of the quark interaction.

2.3 Gap equation solutions

We solve the gap equations for stellar matter under the constraints of the previous subsection. The results are plotted in the Fig. 1 for three different strengths η of the coupling constant in the diquark channel at zero temperature. For the commonly used value of $\eta = 0.75$ the solutions with Lorentzian and NJL formfactors show a normal quark matter phase at intermediate densities and a superconducting phase shifted to higher densities. The solution with Gaussian formfactor exhibits no diquark condensation at all. For $\eta = 1$ superconductivity appears immediately at the chiral restoration with large gaps for the three formfactors. However, in this case a mixed phase construction is necessary to neutralize the matter (see next section and Fig. 2). For an even stronger diquark coupling constant, $\eta = 1.2$, a pure superconducting phase with large diquark gaps occurs and the threshold for the chiral restoration transition is shifted to lower densities than for $\eta = 0.75$ and $\eta = 1.0$ for the three formfactors.

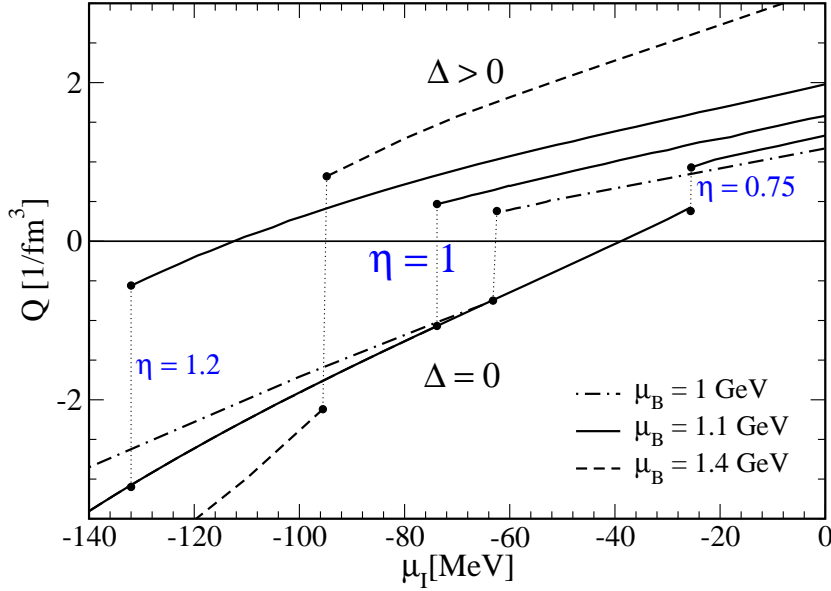


Fig. 2. Electric charge density for 2SC ($\Delta > 0$) and normal ($\Delta = 0$) quark matter phases as a function of isospin chemical potential μ_I along lines of fixed baryon chemical potential μ_B (see legend inside figure) for three different coupling constants: $\eta = 0.75$ (with the jump on the right), $\eta = 1$ (jumps in the middle) and $\eta = 1.2$ (jumps on the left). Gaussian formfactor and $T = 0$ are considered. See text for further explanation.

2.4 Mixed phase construction

To achieve the charge neutrality condition we apply a mixed phase construction between the subphase with diquark condensation and the subphase of normal quark matter using the Gibbs conditions for phase equilibrium. The temperature T , the chemical potentials $\{\mu_i\}$ and the pressure P should be equal for the subphases involved, in particular,

$$P_{\Delta=0}(\mu_B, \mu_I, \mu_e, T) = P_{\Delta>0}(\mu_B, \mu_I, \mu_e, T) \quad (21)$$

where the subscripts denote the normal ($\Delta = 0$) and the superconducting ($\Delta > 0$) subphases.

In the 2SC mixed phase we construct, the charge neutrality is satisfied as a global constraint: the positively charge superconducting phase coexists with the negatively charged normal quark matter phase in an homogeneous mixture. Surface tension and Coulomb effects are calculated to be small enough not to affect the results qualitatively (up to a value of 2 MeV/fm³ on the pressure [16]) and we disregard them in this work.

The charges of the subphases (states corresponding to the local minima in the thermodynamic potential, i.e. solutions of the gap equations) are shown in Fig. 2 for selected values of μ_B as a function of μ_I for the Gaussian formfactor at zero temperature. The upper (lower) branch corresponds to the subphase with (without) diquark condensation, the jump represents the transition between the subphases where the end points are states with the same pressure. For a given μ_B , we can see that increasing the asymmetry $|\mu_I|$ in the system (the mismatch between the up and down quark Fermi seas) the diquark condensation is disfavored. When comparing the charges for different η , we found that for $\eta = 1$ (three lines with jumps that intersect the $Q = 0$ line), the superconducting branches are positively charged and the normal ones are negatively charged. At $Q = 0$ we obtain the coexistence of the subphases at the same pressure and the occurrence of a mixed phase. For $\eta = 0.75$, (at fixed $\mu_B = 1.1$ MeV) the lower branch crosses the $Q = 0$ line and neutral quark matter is just normal. At the same μ_B but in case of the very strong coupling constant $\eta = 1.2$ we obtain charge neutrality for a pure superconducting phase.

To explore the relation between asymmetry and β -equilibrium in neutral matter we plot in Fig. 3 the solution of the gap equations under the condition $Q = 0$ in the μ_I vs. μ_e plane for the Gaussian formfactor at $T = 0$. The results are shown for different values of η and the lines for the β -equilibrium (16) correspond to different values of the chemical potential for the neutrinos and antineutrinos. For a fixed η , e.g. $\eta = 1$, two branches are shown: states in a phase with diquark condensation on the right ($\Delta > 0$, for large

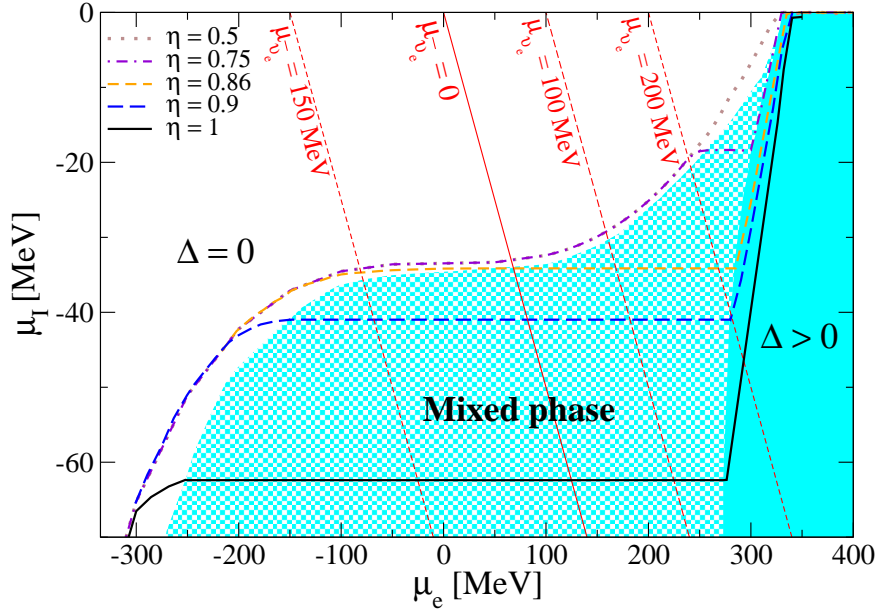


Fig. 3. Solutions of the gap equations and the charge neutrality condition (thick lines crossing the figure) in the μ_I vs. μ_e plane for fixed $\mu_B = 1$ GeV and different values of η with the Gaussian formfactor at $T = 0$. At fixed η , two branches are shown: states with diquark condensation on the right (grey area, $\Delta > 0$) and states of normal quark matter (white area, $\Delta = 0$) on the left. The hatched region in between corresponds to a mixed phase of normal and superconducting quark matter. The lines corresponding to β -equilibrium condition are also shown (solid and dashed thin vertical lines) for different values of the neutrino and antineutrino chemical potential. Stellar matter should satisfy both conditions (intersection of the corresponding lines). If $\mu_{\nu_e} = 0$, a mixed phase is preferable for $\eta \geq 0.86$. For lower values of the coupling constant diquark condensation is not possible in compact stars.

μ_e) and states in a normal quark matter phase ($\Delta = 0$, for large negative μ_e) on the left. The plateau in between them shows that the system undergoes a phase transition from the superconducting to the normal phase (and the two subphases coexist in a mixed phase) when the asymmetry increases or the number of electrons decreases. The stellar matter in compact objects should satisfy both conditions which are simultaneously fulfilled at the intersection of the corresponding lines. Therefore, for the situation in which there are no trapped leptons in the star ($\mu_{\nu_e} = 0$), a mixed phase is preferable for $\eta \geq 0.86$ for the Gaussian formfactor. For lower values of the coupling constant diquark condensation is not possible. A similar analysis shows, for the two other formfactors, that the values of η at which superconductivity is possible are smaller but its appearance is shifted to higher densities, see Fig. 4.

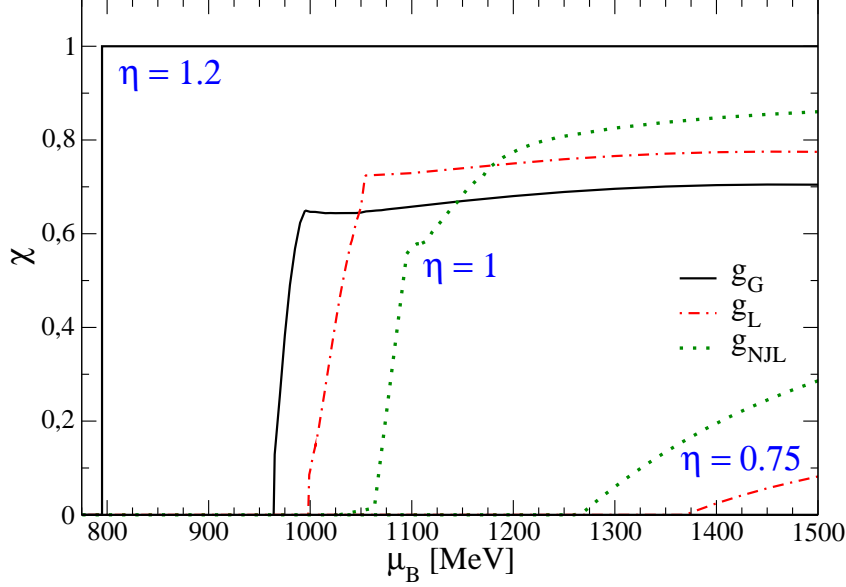


Fig. 4. Volume fraction χ of the 2SC phase as a function of the baryon chemical potential obtained by a Glendenning construction for the charged-neutral NQ-2SC mixed phase. Results are shown for three different diquark coupling constant η and three formfactors of the quark interaction at $T = 0$.

The volume fraction that is occupied by the subphase with diquark condensation is defined as

$$\chi = Q_{\Delta>0}/(Q_{\Delta>0} - Q_{\Delta=0}) \quad (22)$$

and is plotted in Fig. 4 as a function of μ_B at $T = 0$. For a strong coupling like $\eta = 1.2$, we obtain that $\chi = 1$ once quark matter appears and therefore a pure superconducting phase for the three formfactors is realized. For intermediate strength of $\eta = 1$, the fraction of the superconducting subphase is between 0.6 and 0.9 depending on the formfactor and on the density. For $\eta = 0.75$, we obtain that no diquark condensation is possible for the Gaussian formfactor and for the Lorentzian and NJL formfactors the percentage of superconducting matter is less than 50%, whereby the onset of color superconductivity is shifted to higher baryon chemical potentials and thus possibly becomes irrelevant for compact stars.

In Fig. 5 the contribution of the leptons to the volume fraction is shown. While muons are negatively charged and act in favor of the charge neutrality - increasing the fraction of diquark condensation - trapped antineutrinos increment the asymmetry in the system and produce the opposite effect.

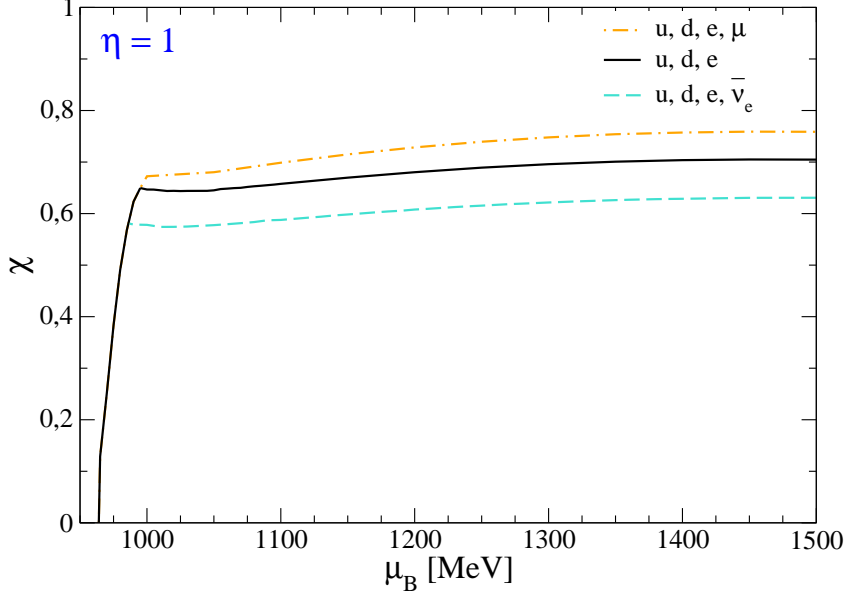


Fig. 5. Same as figure 4 but here the Gaussian formfactor is used to analyze the contribution of the leptons participating in the β -equilibrium for $\eta = 1$. The solid line corresponds to a system of u, d quarks and electrons e . The dash-dotted line shows the increase of χ if also muons μ are included and the dashed line shows the reduction of χ if trapped antineutrinos are considered with $\mu_{\bar{\nu}_e} = 200$ MeV.

2.5 Phase diagram

We explore the relevant region of the phase diagram of QCD for the Gaussian formfactor of our model. For strong coupling constant $\eta = 1$, a rich structure of color superconducting quark matter phases is found above the first order chiral transition (solid line on the left) as it is shown in Fig. 6. For the quark matter sector we obtain that at low temperatures (up to $25 \div 35$ MeV depending on μ_B) the mixed phase NQ-2SC is energetically favored with increasing volume fraction χ of the 2SC phase as the temperature increases (solid lines characterize equal values of χ indicated over the lines). At intermediate temperatures a large region of pure 2SC phase is obtained. At higher temperatures a narrow window of g2SC occurs before the second order phase transition to a deconfined phase takes place. Nevertheless, for weak and intermediate coupling constants (e.g. for $\eta = 0.75$ MeV) this structure of superconducting phases disappears and the normal quark matter phase is preferable.

For higher values of the chemical potential, $\mu_B \geq 1500$ MeV, we expect the strange quark to appear and CFL phase to dominate (see [12] for a recent three flavor phase diagram with a similar parametrization within an NJL model).

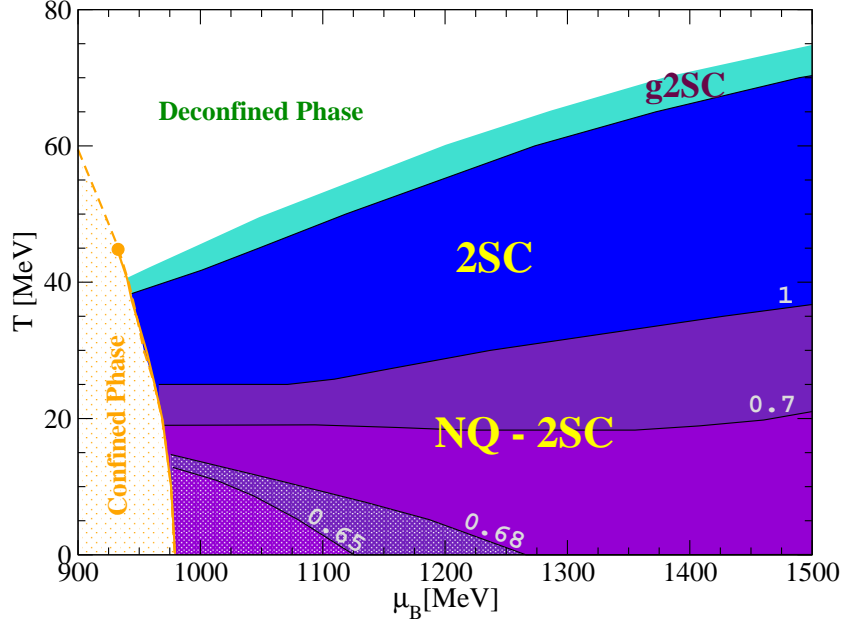


Fig. 6. Relevant region of the phase diagram for compact stars applications for two flavor neutral quark matter within a nonlocal chiral model with Gaussian formfactor. For strong coupling constant $\eta = 1$, a rich structure of color superconducting quark matter phases is found above the first order chiral transition (solid line on the left). For the quark matter sector we obtain the occurrence of a mixed phase NQ-2SC, a pure 2SC, a g2SC and a second order phase transition to a deconfined phase as the temperature increases. Solid lines within the NQ-2SC region characterize equal values of the volume fraction χ of the 2SC phase indicated by numbers over the corresponding lines. For $\eta = 0.75$ this structure of superconducting phases disappears and we obtain normal quark matter above the chiral phase transition.

3 Quark matter equation of state

The number density n_j of the particle species j in the mixed phase are given by

$$n_j = \chi n_{j\Delta>0} + (1 - \chi) n_{j\Delta=0} , \quad (23)$$

and shown as functions of μ_B at zero temperature in Fig. 7. From these graphs we can see that increasing η the number of electrons in the system increases by order of magnitudes and the onset of the phase transition from the vacuum is shifted to lower densities. The energy density ε in the mixed phase is given in a similar form as

$$\varepsilon = \chi \varepsilon_{\Delta>0} + (1 - \chi) \varepsilon_{\Delta=0} . \quad (24)$$

We evaluate the quark matter EoS at $T = 0$ within this nonlocal chiral model and show the results in Fig. 8.

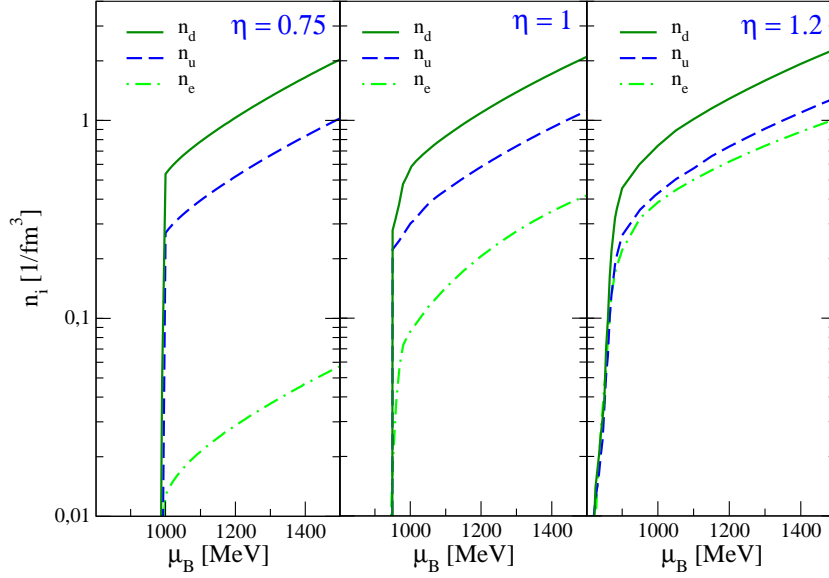


Fig. 7. Quark and electron number densities as a function of the baryon chemical potential at $T = 0$. The three panels show results for different coupling constants of the diquark interaction, $\eta = 0.75, 1.0, 1.2$.

As in [17] we can display the results for the pressure in the form of a bag model

$$P = P_{id}(\mu_B) - B(\mu_B) , \quad (25)$$

where $P_{id}(\mu_B)$ is the ideal gas pressure of quarks and $B(\mu_B)$ a *density dependent* bag pressure, see Fig. 9. For $\eta = 0.75$, $B \simeq 75 \text{ MeV/fm}^3$ is nearly a constant function of the density; for $\eta = 1$ we found a large pressure effect of the diquark condensate and therefore a lower and strongly density dependent B . Finally, for $\eta = 1.2$ the pressure of the condensate is so huge that the bag pressure reaches negatives values.

4 Conclusion

Within our study of the equation of state and phase structure for two-flavor quark matter at low temperature under compact star constraints we found that the occurrence of a 2SC phase is sensitive to variations of both the formfactor

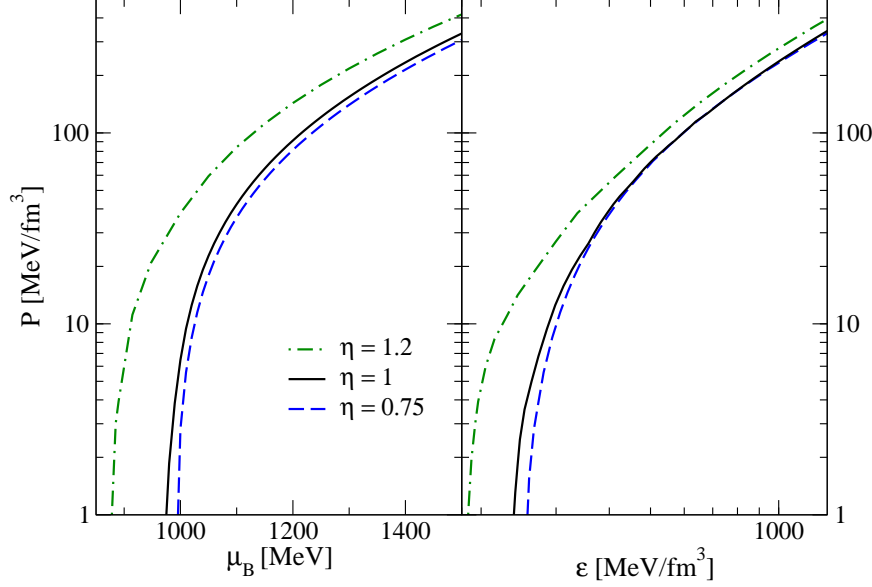


Fig. 8. EoS for strongly interacting matter at zero temperature under compact star constraints for three coupling parameters $\eta = 0.75, 1.0, 1.2$. Left panel: pressure vs. chemical potential; right panel: pressure vs. energy density.

Formfactor	$\Lambda[\text{GeV}]$	$G_1 \Lambda^2$	$m[\text{MeV}]$
Gaussian	1.025	3.7805	2.41
Lorentzian	0.8937	2.436	2.34
NJL	0.9	1.944	5.1

Table 1

Parameter sets ($\Lambda, G_1 \Lambda^2, m$) of the nonlocal chiral quark model for the different formfactors.

of the nonlocal quark interaction and the strength of the diquark coupling constant. Our results suggest that for standard values of the coupling $0.5 < \eta < 0.75$ either the 2SC phase does not occur (Gaussian formfactor) or it exists only in a mixed phase with normal quark matter with a volume fraction less than 20 – 40 %, occuring at high baryon chemical potentials $\mu_B > 1200$ MeV and most likely not relevant for compact stars. We obtain that neither the g2SC phase occurs in compact stars; it appears in a narrow window at higher temperatures and for strong coupling constants (e.g. $\eta = 1$).

This result suggests that other pairing patterns like the CSL phase with rela-

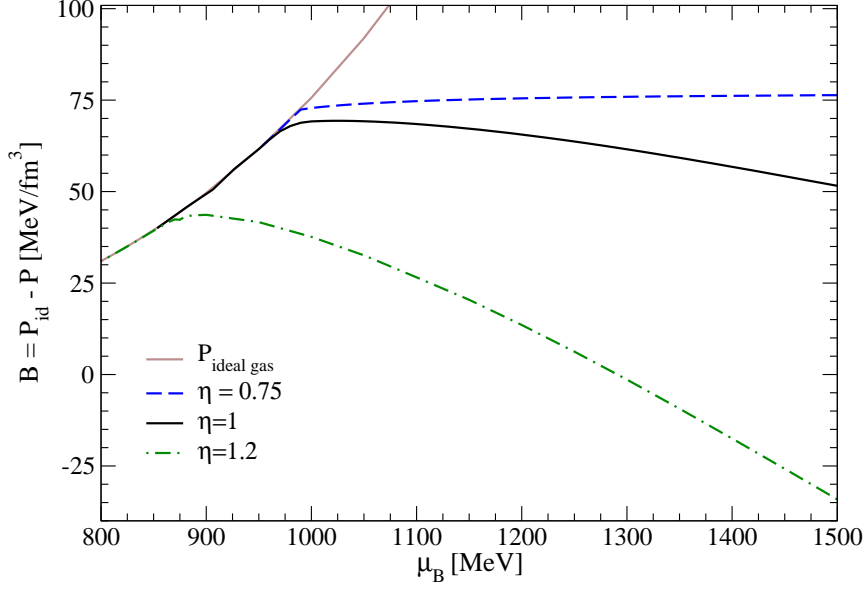


Fig. 9. Bag pressure - as defined in [17] - for different diquark coupling constants $\eta = 0.75, 1.0, 1.2$ in dependence on the baryon chemical potential for Gaussian formfactor at $T = 0$.

tively small gaps of the order of $10 - 100$ keV may be realized. This hypothesis is in accord with a recent description of modern cooling data using hybrid star models.

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